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Dynamic investigation of hard viscoelastic materials by ball bouncing experiments

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An experimental device is described for studying the behaviour of a material subjected to the impact of a rigid spherical indentor, in the temperature range −25 to +90° C. For each experimental run performed on a quasi-semi-infinitely hard viscoelastic PVC material, the coefficient of restitution, the duration of impact and maximum penetration were measured. From a series of runs performed with different impact velocities, the effective damping was computed as well as the dynamic hardness. Comparison with a periodic test method proved to be fairly good. In particular the same results on specific losses were obtained by both methods, as long as the impact kinetic energy did not exceed 170 mJ. Furthermore an estimate of the storage E modulus can be made from the contact time. As this paper deals with linear viscoelasticity, the specific deformation energies involved are not too high.

List of symbols

a = radius of the contact spherical cap, mm
E_1, E_2 = complex moduli, hbar
e = coefficient of restitution
M = mass of the projectile, g
R = radius of the spherical indentor, mm
T = temperature, ° C
T_g = glass transition temperature, ° C
t_c = contact time, msec
V_i = impact velocity, cm sec^{-1}
V_R = rebound velocity, cm sec^{-1}
x = penetration depth, mm.
$\mu(t)$ = relaxation function, hbar
$\mu^{-1}(t)$ = creep function
ν = Poisson's ratio
ω = Angular frequency, rad sec^{-1}

1. Introduction

Because synthetic macromolecular compounds are becoming more and more numerous and their behaviour is different, it is quite difficult to define a single mechanical test for classifying a specific compound. Some investigators [1–4] have pointed out the possibility of using the ball bouncing test, particularly for analysing the dynamic hardness of viscoelastic materials [5]. Indeed because the strain-rates and impact duration involved in this kind of test are respectively very high and brief, resilience and recovery both play a part.

For viscoelastic materials, the most satisfactory definition is the ratio of the dissipative energy to the maximum deformed volume

$$P_{dyn} = \frac{\Delta W}{V_{max}}.$$

The dissipation of energy is calculated from the impact velocity and the coefficient of restitution

$$\Delta W = \tfrac{1}{2}M(V_i^2 - V_R^2) = \tfrac{1}{2}MV_i^2(1 - e^2).$$

The average value of the deformed volume should be seriously considered, but experimental difficulties are then encountered. Therefore, the maximum volume under the contact cap arbitrarily, at the end of the loading phase must be used:

$$V_{max} = \pi R\left(\frac{x}{2}\right)^2.$$

Under these conditions the dynamic hardness is

$$P_{dyn} = 2MV_i^2(1-e^2)/\pi R x_{max}^2.$$

2. Theoretical analysis

The same assumptions as stated in the study of the Hertzian elastic impact [6], apply, i.e.:

- a small rigid indentor (Love's criterion)
- the sample can be considered as a semi-infinite half-space medium (Hunter's criterion)
- penetration is slight compared with the size of the spherical punch (Hertz's criterion)
- low impact velocity (Boltzmann's criterion).

With these assumptions the problem is a quasi-steady-state one and the equation of motion can be written for an elastic impact:

$$Mx'' + \frac{8\sqrt{R\mu}}{3(1-\nu)} x^{3/2} = 0. \quad (2)$$

In transposing this equation for the case of a viscoelastic material, by using the Riemann-Stieljes integrals, a difficulty arises due to the viscoelastic behaviour and because in the case we are studying, the contact radius $a(t)$ reaches a maximum, i.e. the boundary conditions are not monotonic. Two periods must then be distinguished: the loading phase and the unloading phase.

An important complementary assumption is used: the Poisson ratio is assumed to be constant, which appears reasonable on the basis of the short impact duration and the glassy physical state of the material. The governing equations can then be written starting from $E_{q(2)}$ [7]:

(1) *for the loading period*

$$t < t_m \quad \frac{d^2x}{dt^2} = -\frac{8}{3M(1-\nu)R} \cdot \int_0^t \mu(t-t')\,da^3(t') \quad (3)$$

and

$$R \cdot x(t) = a^2(t), \quad (4)$$

(2) *for the withdrawal period*

$$t > t_m \quad \frac{d^2x}{dt^2} = -\frac{8}{3M(1-\nu)R} \int_0^{t_1(t)} \mu(t-t')\,da^3(t') \quad (5)$$

and

$$R \cdot x(t) = a^2(t) - \int_0^t \mu^{-1}(t-t') \frac{d}{dt'} \left[\int_{t_1(t')}^{t'} \mu(t'-t'')\,d(a^2(t'')) \right] dt' \quad (6)$$

the $t_1(t)$ function is defined by

$$a(t_1) = a(t).$$

A complete solution of the impact problem is available only through numerical tools. Functions $\mu(t)$ and $\mu^{-1}(t)$ are chosen in order to obtain a good fit between numerical and experimental data. As a simplification, we can assume that the material's behaviour is described by a simple Maxwell-solid model. This assumption will be proved to be valid for the material investigated.

Let τ be the relaxation time of the model. Hence the relaxation function will be:

$$\mu(t) = \mu_0 \exp(-t/\tau), \quad (8)$$

while the creep function is

$$\mu^{-1}(t) = \mu_0^{-1}(1 + t/\tau). \quad (9)$$

From Equations 3 and 5 it can be deduced that for $t_c \ll \tau$:

$$1 - e = \tfrac{4}{9}(t_c/\tau) \quad (10)$$

$$x_{max} = \frac{V_i t_c}{2.94} [1 + 0.195(1-e)]. \quad (11)$$

Then for a Maxwell solid the dynamic hardness can be written using Equations 1, 10 and 11

$$P_{dyn} = \frac{2M(2.94)^2}{\pi R t_c^2} \cdot \frac{1-e^2}{1+0.390(1-e)}. \quad (12)$$

3. Experimental device and operating technique

The material studied was a Dupont De Nemours PVC and the indentor was made of tungsten carbide. The sizes of indentor and PVC sample were chosen so as to satisfy the theoretical assumption previously stated. The radius of the spherical indentor was 6.75 mm, and the PVC sample was cylindrical in shape (thickness 40 mm and diameter 100 mm). The moving indentor was set on a steel block which was sustained in air and able to run along a rail (Fig. 1); the whole mass was 340 g. This block was pushed away at the start by the sudden release of a coil spring the compression of which was altered, by an electromagnet, to achieve the desired impact velocity. The indentor's velocity was computed from the time taken by the steel block to run past a logical output phototransistor connected to an intervalometer. The PVC sample was attached to a heavy concrete mass (about 90 kg).

So that the plastic sample could be used for several runs, the impact point was chosen outside of the axis of the sample. The distance between them was about 20 mm, and after each run a 10° rotation of the sample was made. The overall impact duration was measured electrically. A thin Cu–Ni alloy layer sprayed on cathodically was placed at the +5 V potential of an electric timer while the indentor was connected to the ground circuit. Penetration depth during the test was continuously measured with a pair of contactless inductive proximity transducers operating with 100 kHz carrier frequency. Measurements were made with one of the transducers, while the other was used solely to adjust the cross voltage. A variation in the distance between the operating transducer and a high-permeability coil cemented to the moving block causes a change in the coil's inductivity and then a disequilibrium of the Wheatstone bridge. A demodulation is performed which is recorded, after amplification, on a storage oscilloscope.

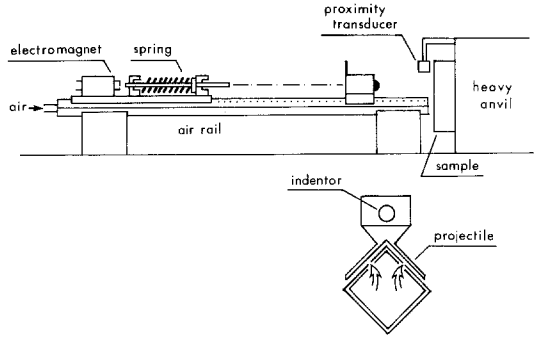

Figure 1 Arrangement of the experimental device for the ball bouncing experiment.

The temperature of the sample is controlled by the circulation of a thermostatically controlled fluid (cold gaseous nitrogen or hot oil) around both the plastic sample and its attaching device. With this technique the temperature can be maintained in the range −30 to +110° C.

4. Experimental results

4.1. Contact time

As shown in Fig. 2, for a given temperature a log linear correlation is observed between V_i and t_c, with the slope of the line being equal to 0.20. For both the temperature and velocity ranges investigated, it may, therefore, be concluded that the tested PVC follows the elastic impact law:

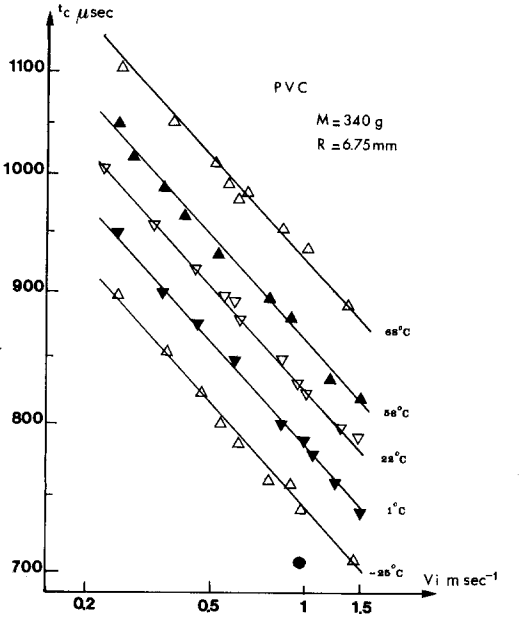

Figure 2 Measured contact time plotted against impact velocity for five different temperatures of the material.

$$t_c = k \cdot V_i^{-0.2}, \quad k = 2.94 \left[\frac{32\sqrt{R}}{15M(1-\nu^2)} E \right]^{2/5} \tag{13}$$

E being the Young's modulus.

Assuming the existence of an equivalent frequency ω for slightly viscoelastic materials ($\omega = \Pi/t_c$), the dynamic storage modulus $E_1(\omega)$ can be defined from t_c by

$$E_1(\omega) = \frac{15M(1-\nu^2)}{32R^{1/2}} \left[\frac{t_c V_i^{1/5}}{2.94} \right]^{-5/2}. \tag{14}$$

With this assumption, a rough estimate of the elastic part of the Young's modulus appears to be possible from a contact time measurement, which is quite interesting due to the simplicity and rapidity of the measurement [8].

The isothermal curves plotted in Fig. 2 show an extensive modification of the variation of contact time versus velocity when the temperature rises to near $T_g = 68°$ C which is the glassy state transition temperature level.

4.2. Coefficient of restitution

For a given impact velocity, a very slight variation of the coefficient of restitution with temperature, in the glassy state, is observed in Fig. 3, while a continuous fall can be observed from a temperature of approximately 70° C: the PVC then begins to behave as a rubber-like material.

From the value of the coefficient of restitution,

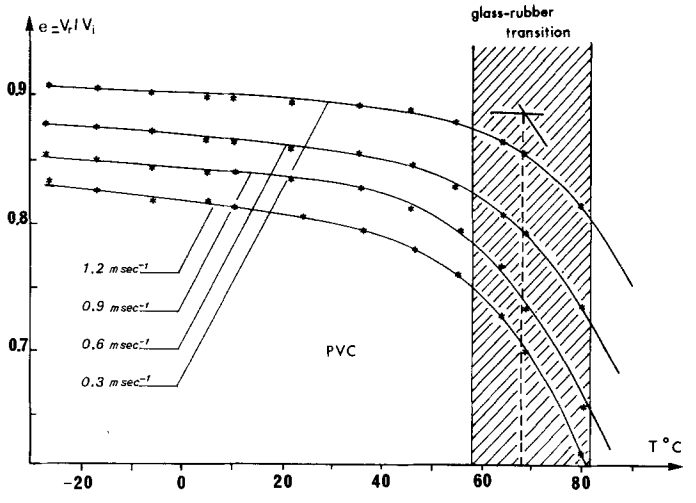

Figure 3 Coefficients of restitution as a function of the temperature for four different impact velocities.

e, an estimate is possible for the material losses. Indeed the ratio between the mechanical energy lost ΔW, and the elastic energy recovered, W, is given by:

$$\frac{\Delta W}{W} = \frac{\frac{1}{2}M(V_i^2 - V_R^2)}{\frac{1}{2}MV_R^2} = \frac{1-e^2}{e^2}. \quad (15)$$

A comparison was made between the results given by this formula and those resulting from a harmonic test

$$\frac{\Delta W}{W} = \Pi \tan \delta \quad (16)$$

where δ is the loss angle of the tested material.

TABLE I

T(° C)	V_i (m sec^{-1})	$\frac{1-e^2}{e^2}$	$\Pi \tan \delta$
20	0.45	0.13	0.15
	0.56	0.15	
56	0.35	0.13	0.17
	0.52	0.16	
70	0.37	0.33	0.22
	0.46	0.40	

Table I shows good agreement between the two ways of determining the damping capacity tan (δ) for the glassy state as long as the impact velocity does not exceed 0.5 m sec^{-1}. This result appears quite useful because of the great ease in recording the results of an impact test.

4.3. Maximum penetration depth and dynamic hardness

A comparison was made between the experimental values for the maximum penetration depth and theoretical values calculated using Equation 11 from the coefficient of restitution and contact time. As long as the temperature is lower than T_g, very good agreement is observed between the two sets of values. However a small shift is observed between the two sets of results (Fig. 4). For determining distances between the high-μ coil on the moving block and the transducer, it was not possible to obtain the exact relative initial position between coil and transducer for both the standardization procedure and the experimental runs. Hence a determination error exists which is constant as long as temperature and sample are not changed.

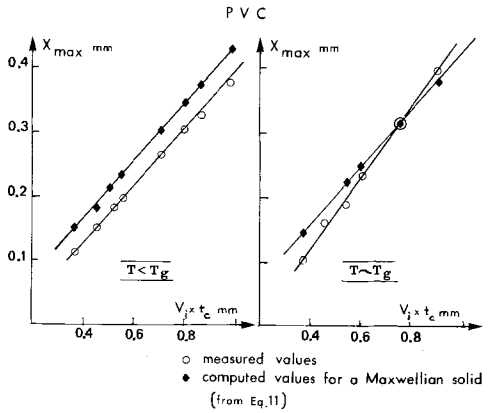

Figure 4 Maximum penetration depth plotted against $V_i \times t_c$, below and up above the glass temperature level.

For a given temperature, all the runs were performed with the same sample, which was only turned round on its axis after each run. The error is, therefore, the same for a series of experiments.

In spite of the experimental error just described, the experimental results enable us to con-

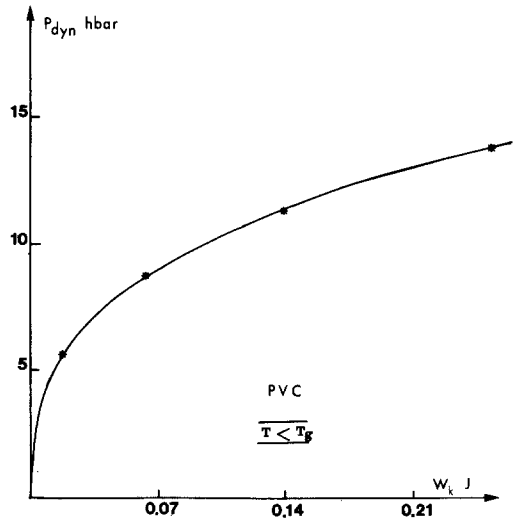

Figure 5 Values of the dynamic hardness plotted against the kinetic energy of the projectile.

clude that the mechanical behaviour of the PVC samples studied can be described by a simple Maxwell model, but only for the glassy state. Equation (12) can, therefore, be used to estimate the dynamic hardness as a function of the kinetic impact energy (Fig. 5). For the impact velocity range explored the dynamic hardness of PVC was found, in the glassy state, to be independent of temperatures. When the PVC exhibits a rubber-like behaviour, i.e. for $T > T_g$, Equation 1 has to be used directly. Experimental determination of penetration depth must then be as thorough as possible for the effective determination of dynamic hardness.

5. Conclusion

In conclusion it must be emphasized that the impact test can be used as an efficient and quick method to experimentally evaluate the mechanical characteristics of linear viscoelastic materials: i.e. (1) damping capacity, (2) dynamic storage modulus and (3) dynamic hardness.

In the case of materials obeying a Maxwell model, the determination of dynamic hardness is straightforward from two sets of experimental data: i.e. the contact time and the coefficient of restitution.

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